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# Towards a realistic picture of CP violation in heterotic string models

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Abstract: We find that dilaton dominated supersymmetry breaking and spontaneous CP violation can be achieved in heterotic string models with superpotentials singular at the fixed points of the modular group. A semi-realistic picture of CP violation emerges in such models: the CKM phase appears due to a complex VEV of the $T$-modulus, while the soft supersymmetric CP phases are absent due to an axionic-type symmetry.

Keywords: Superstrings and Heterotic Strings, Supersymmetry Breaking, Supersymmetry Phenomenology, CP violation.



 http://jhep.sissa.it/archive/papers/jhep082002007/jhep082002007.pdf

## Contents



## 1. Introduction

While string theory remains an excellent candidate for the theory of everything, its connection to the presently observed world remains obscure. In this letter, we attempt to bridge one of the gaps, that is to address the problem of CP violation. Recent observations have shown that CP is heavily violated in the CKM (Cabibbo-Kobayashi-Maskawa) mixing matrix [1]. On the other hand, if we are to retain low energy supersymmetry as a solution to the hierarchy problem, the electric dipole moment (EDM) experiments [2] require the soft SUSY CP-phases to be vanishingly small (see [3] for a recent review). Thus, the challenge is to find a supersymmetric string model which produces a large CKM phase while having small enough soft SUSY CP-phases. The problem is exacerbated by the fact that even if the soft CP-violating phases are absent initially, they are often induced by a quark superfield basis rotation [4].

It is well known that CP is a gauge symmetry in string theory [5] and therefore must be broken spontaneously. Natural candidates for breaking CP are the dilaton ($S$) and moduli[1] ($T$) fields [6] which are common to string models. The former, however, cannot produce the CKM phase, so a complex $\langle T\rangle$ is required in realistic models. Of course, CP violation may originate from an entirely different sector, but this would be highly model-dependent and so we do not discuss this possibility here.

In what follows, we will concentrate on string models possessing target space modular invariance, such as heterotic orbifolds. That is, physics is invariant under the $\mathrm{PSL}(2,Z)$ transformations

$$T \longrightarrow \frac{aT-ib}{icT+d}\,, \tag{1.1}$$

$$S \longrightarrow S+\frac{3}{4\pi^2}\delta_{GS}\ln(icT+d)\,, \tag{1.2}$$

[1]There are three such moduli in most models, one for each complex plane, but here we will assume that they all take the same value.

where $a, b, c, d$ are integers obeying $ad - bc = 1$ and $\delta_{GS}$ is the Green-Schwarz anomaly cancellation coefficient. This symmetry imposes strict constraints on the form of the effective superpotential and plays a crucial role in our analysis. As in our earlier studies [7], we impose the following phenomenological constraints:

1. the dilaton is stabilized at $\mathrm{Re}S \sim 2$
2. the pattern of CP violation is phenomenologically acceptable
3. a realistic SUSY breaking scale

Previous attempts to produce CP violation [8] did not address the problem of dilaton stabilization and thus were not fully realistic. Additional constraints such as the absence of the flavor changing neutral currents (FCNC) can also be imposed, but these are often safisfied automatically in this class of models if a non-trivial CKM phase is produced at the renormalizable level (due to the Yukawa coupling selection rules) [7].

Dilaton stabilization has an important effect on the pattern of supersymmetry breaking. Our previous studies [7] have shown that it often forces moduli-dominated supersymmetry breaking which has a disastrous phenomenology. On the other hand, when dilaton-dominated SUSY breaking was produced, no CP violation appeared.

In the present letter, we will try to reconcile CP violation and correct supersymmetry breaking by relaxing the assumption that the superpotential has no singularities in the fundamental domain of the modular group. The singularities appear at the points in the moduli space where the threshold corrections to the gauge couplings become infinite. This, of course, happens at $\mathrm{Re}T \to \infty$ corresponding to a large contribution from light Kaluza-Klein states, but may also occur at other points in the moduli space [9]. Since explicit examples exhibiting this behavior are lacking, we will take the *bottom-up* approach, i.e. adopt the above phenomenological requirements as our strarting point while being consistent with the modular invariance. We find that singular superpotentials allow for phenomenologically interesting minima at which CP violation is present in the Standard Model sector but not in the soft SUSY breaking terms.

We shall proceed as follows. In the next section, we present our framework. In section 3, we discuss patterns of the minima of the scalar potential and provide examples of dilaton dominated supersymmetry breaking with a reasonable pattern of CP violation.

## 2. Framework

Heterotic string models often contain a "hidden" sector, i.e. a sector which does not have direct non-gravitational interactions with the Standard Model fields. Then it is quite plausible that supersymmetry breaking occurs in this sector and is communicated to the visible sector by gravity. One of the popular schemes to break supersymmetry in the hidden sector employs gaugino condensation [10]. This possibility is quite attractive since a hierarchy between the Planck and SUSY breaking scales is created dynamically through a dimensional transmutation. In this paper, we assume that gaugino condensation is indeed realized although our discussion often applies more generally and is restricted by the target space modular invariance only.



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Gaugino condensation can be realized in the $E_8 \otimes E_8$ heterotic string theory where the condensate lives in one $E_8$, the other forming the observable sector. After intergrating out the condensate and any matter fields ($M$ generations transforming in SU($N$)) by using a truncated approximation, the Veneziano-Yankielowicz superpotential which describes the condensate is given by [11, 12]:[2]

$$W = \tilde{d}\frac{e^{\frac{-3S}{2\tilde{\beta}}}}{\eta(T)^{6-\frac{9\delta_{GS}}{4\pi^2\tilde{\beta}}}} \tag{2.1}$$

where $\tilde{\beta} = \frac{3N-M}{16\pi^2}$ is the beta function and $\tilde{d} = (M/3 - N)(32\pi^2 e)^{\frac{3(M-N)}{3N-M}}(M/3)^{\frac{M}{3N-M}}$. The Kähler potential for the dilaton and moduli is [13]:

$$K = -\ln Y - 3\ln(T + \overline{T})\,, \tag{2.2}$$

where a modular invariant combination $Y$ is given by $Y = S + \overline{S} + \frac{3}{4\pi^2}\delta_{GS}\ln(T + \overline{T})$. The consequent scalar potential is calculated via the supergravity relation

$$V = e^G\left(G_i\left(G^i_j\right)^{-1}G^j - 3\right), \tag{2.3}$$

where $G = K + \ln(|W|^2)$ and the indices denote differentiation. The sum runs over the fields in the system ($S$ and $T$ in our case). Supersymmetry is broken by VEVs of the auxiliary fields ($j = S, T$):

$$F_j = e^{G/2}\left(G^i_j\right)^{-1}G_i\,, \tag{2.4}$$

The superpotential describing a single gaugino condensation does not lead to dilaton stabilization. Thus one has to consider modifications of either the superpotential or the Kähler potential. Some common choices are[3] (1) to employ multiple gaugino condensates [15], (2) to postulate S-duality [16], or (3) to incorporate non-perturbative corrections to the Kähler potential [17]. The first two options typically lead to moduli-dominated SUSY breaking which entails a number of phenomenological problems [7]. The third possibility is known to produce dilaton domination (at least when the issues of CP violation are not addressed), so we will choose this last option. The non-perturbative Kähler potential is assumed to be of the form [17]

$$K_S = \ln\left(\frac{1}{Y} + d\left(\frac{Y}{2}\right)^{\frac{p}{2}}e^{-b\sqrt{\frac{Y}{2}}}\right), \tag{2.5}$$

where $d, p, b$ are certain constants (with $p, b > 0$). This form is based on the requirements that the non-perturbative corrections vanish in the weak coupling limit $S \to \infty$ and that they are zero to any order in a perturbative expansion in $1/Y$. Dilaton stabilization with this type of the Kähler potential has been studied in detail in ref. [18] with the result that an acceptable SUSY breaking scale and the dilaton value can be obtained, whereas in physical cases ($K^S_S > 0$) the cosmological constant does not vanish.

[2] We assume that the Kac-Moody level of the gauge group is one.

[3] In the context of type I string models, see also [14].



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Minimization of the scalar potential derived from the superpotential (2.1) yields CP-conserving values of the modulus. To obtain CP violation, the superpotential must be modified. In particular, it can be multiplied by a modular invariant function $H(T)$ [9, 8]:

$$W \to W \times H(T)\,, \tag{2.6}$$

where

$$H(T) = \Big[j(T) - 1728\Big]^{\frac{m}{2}} j(T)^{\frac{n}{3}} P\left[j(T)\right] \tag{2.7}$$

with $j(T)$ being the absolute modular invariant function (see [9] for an explicit expression) and $P[j(T)]$ being some polynomial. To avoid singularities in the fundamental domain $m$ and $n$ have to be positive integers. Although this generalized superpotential is consistent with the modular symmetry, explicit examples of the threshold corrections leading to this superpotential are lacking, although the modular invariant function $j(T)$ does appear in explicit calculations [19].

Dilaton stabilization and CP violation in models with the generalized superpotential were studied in ref. [7]. No phenomenologically acceptable minima were found. However, this analysis was based on the assumptions that $m$ and $n$ were positive. In general, this is not necessarily true [9] and the superpotential may have singularities at certain points in the moduli space. In our present analysis, we allow for singularities at the fixed points of the modular group and find that this possibility is much more attractive phenomenologically.

Let us now list the relevant supersymmetry breaking terms. The soft SUSY breaking lagrangian in the visible sector is

$$\mathcal{L}_{\rm soft} = \frac{1}{2}\left(M_a \lambda^a \lambda^a + \text{h.c.}\right) - m^2_\alpha \hat{\phi}^{*\alpha}\hat{\phi}^\alpha - \left(\frac{1}{6} A_{\alpha\beta\gamma}\hat{Y}_{\alpha\beta\gamma}\hat{\phi}^\alpha\hat{\phi}^\beta\hat{\phi}^\gamma + B\hat{\mu}\hat{H}_1\hat{H}_2 + \text{h.c.}\right), \tag{2.8}$$

where $\hat{Y}_{\alpha\beta\gamma}$ and $\hat{\mu}$ are the Yukawa couplings and the $\mu$-term for the canonically normalized fields $\hat{\phi}$. With the Kähler potential and the superpotential of the form

$$\begin{aligned} K &= \hat{K} + \tilde{K}_\alpha \phi^{*\alpha}\phi^\alpha + \left(Z H_1 H_2 + \text{h.c.}\right), \\ W &= \hat{W} + \frac{1}{6} Y_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma\,, \end{aligned} \tag{2.9}$$

$\hat{Y}_{\alpha\beta\gamma}$ and $\hat{\mu}$ are given by [20]

$$\begin{aligned} \hat{Y}_{\alpha\beta\gamma} &= Y_{\alpha\beta\gamma}\frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2}\left(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{-1/2}, \\ \hat{\mu} &= \left(m_{3/2} Z - \bar{F}^{\bar{m}}\partial_{\bar{m}} Z\right)\left(\tilde{K}_{H_1}\tilde{K}_{H_2}\right)^{-1/2}. \end{aligned} \tag{2.10}$$

Here $m = (S, T)$; $\hat{K}$ and $\hat{W}$ are the hidden sector Kähler potential and superpotential. The Kähler function for a field of modular weight $n_\alpha$ is $K = (T + \overline{T})^{n_\alpha}$. For definiteness, we have assumed the Giudice-Masiero mechanism for generating the $\mu$-term [21]. This requires the presence of a $ZH_1H_2$ term in the Kähler potential, which can be implemented in even order orbifold models possessing at least one complex structure modulus, $U$ (which we will set to 1/2). $Z$ in this case is given by [22]

$$Z = \frac{1}{(T_3+T_3^*)(U_3+U_3^*)}\,, \tag{2.11}$$

The canonically normalized fields are obtained by the rescaling $\hat{\phi}_\alpha = \tilde{K}_\alpha^{1/2}\phi_\alpha$. The gaugino masses, scalar masses, A-terms, and the B-term are expressed, respectively, as [20]:

$$\begin{aligned}
M_a &= \frac{1}{2}(\mathrm{Re} f_a)^{-1} F^m \partial_m f_a\,, \qquad\qquad (2.12)\\
m_\alpha^2 &= m_{3/2}^2 + V_0 - \bar{F}^{\bar{m}} F^n \partial_{\bar{m}}\partial_n \ln \tilde{K}_\alpha\,,\\
A_{\alpha\beta\gamma} &= F^m\left[\hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)\right]\,,\\
B &= \hat{\mu}^{-1}\left(\tilde{K}_{H_1}\tilde{K}_{H_2}\right)^{-\frac{1}{2}}\Big[(2m_{3/2}^2+V_0)Z - m_{3/2}\bar{F}^{\bar{m}}\partial_{\bar{m}}Z +\\
&\qquad + m_{3/2}F^m\left(\partial_m Z - Z\ \partial_m \ln(\tilde{K}_{H_1}\tilde{K}_{H_2})\right) -\\
&\qquad - \bar{F}^{\bar{m}}F^n\Big(\partial_{\bar{m}}\partial_n Z - \partial_{\bar{m}}Z\ \partial_n \ln(\tilde{K}_{H_1}\tilde{K}_{H_2})\Big)\Big]\,.
\end{aligned}$$

Any of these terms can, in general, be complex. However, the EDM measurements require them to have very small CP-phases. This is the notorious SUSY CP problem. In string models, additional difficulties arise because $A_{\alpha\beta\gamma}$ are generically flavor-non-universal and the flavor rotation to the basis where the quark masses are diagonal would induce $O(1)$ soft CP violating phases even if the soft terms were real initially [4]. The problem is exacerbated by the fact that this rotation will produce terms proportional to the masses of the third generation quarks in the diagonal entries of the $A$-terms.

Note that these problems arise when SUSY and CP are broken in the same sector. That is, if the source of the CKM phase (in our case, complex $\langle T\rangle$) also breaks supersymmetry. Although this is a generic situation, we will show that this is not necessarily true and there are many vacua in which $F_T \sim 0$. This would remove the sources of the EDMs due to the non-universality (the "string" CP problem), whereas flavor-universal CP phases may still persist. However, the latter are absent in our case due to the symmetry $S \to S + i\alpha$ which allows us to make the soft terms real.

## 3. Patterns of the Minima

With positive $m, n$, the minima in $T$ often fall at the fixed points of the modular group [7]. At these points the CKM phase vanishes [23][4] and supersymmetry often stays unbroken. In most other cases, the minima are on the unit circle where, again, there is no supersymmetry breaking ($G_S = G_T = 0$). A more realistic (but hard-to-achieve) possibility is when $\langle T\rangle$

[4]If the Standard Model sector exhibited modular invariance, this would also apply to the boundary of the fundamental domain [24]. However, this is not the case in semi-realistic orbifold models [23].

is inside the fundamental domain but close to the fixed point. However, this results in tachyons, large EDMs, and a suppressed CKM phase [7].[5] Clearly, these vacua are not phenomenologically viable.

These problems can be rectified if we allow for negative $m$ and $n$. Indeed, this leads to singularities at the fixed points such that the minimum is "repelled" from them (since $V \sim |W|^2$) and pushed inside the fundamental domain. One should remember that $m$ and $n$ cannot be arbitrarily large (in magnitude) negative numbers if modulus stabilization is to be achieved. The minimum in $T$ is at a finite value if the superpotential diverges at $T \to \infty$ (and at its dual point, $T = 0$). At large $T$,

$$\begin{aligned} \eta(T)^{-1} &\to e^{\pi T/12}\,, \\ j(T) &\to e^{2\pi T}\,, \end{aligned} \tag{3.1}$$

so if the polynomial $P[j(T)]$ is of degree $q$ then the divergence of the superpotential at infinity requires

$$\frac{m}{2} + \frac{n}{3} > -q - \frac{1}{4}\,. \tag{3.2}$$

In the pure Yang-Mills case, there are further restrictions that $H(T)$ have no poles or zeros at infinity. This is because the asymptotic behavior of the superpotential should match that of the threshold corrections and the simple superpotential (2.1), i.e. $H(T) = 1$, does the job [9]. This would require $m/2 + n/3 = -q$ with $q$ being some integer.[6] We have analyzed supersymmetry breaking in such cases and found that they do not lead to a reasonable phenomenology. The problem is that the potential is often minimized at a zero of the polynomial $P[j(T)]$ where supersymmetry remains unbroken.

Inclusion of the matter fields can change the asymptotic behavior of the superpotential [26]. So, we will only require (3.2). For simplicity we will set $P[j(T)] = 1$. In order to repel the minimum from both of the fixed points, both $m$ and $n$ should be negative. Then, $m$ and $n$ have to be rather small in magnitude and fractional (if $H(T)$ is to remain a rational function of $j(T)$). Since $j(T) - 1728 \sim (T-1)^2$ in the proximity of $T = 1$ and $j(T) \sim \left(T - e^{\pm\frac{i\pi}{6}}\right)^3$ around $T = e^{\pm\frac{i\pi}{6}}$, the resulting singularities at the fixed points are

$$\begin{aligned} H(T) &\sim (T-1)^m && \text{at} \quad T \simeq 1\,, \\ H(T) &\sim \left(T - e^{\pm\frac{i\pi}{6}}\right)^n && \text{at} \quad T \simeq e^{\pm\frac{i\pi}{6}}\,. \end{aligned} \tag{3.3}$$

We stress that since it is unclear whether or not these singularities indeed appear in explicit models, we will take a bottom-up approach and assume $m, n$ to be free parameters subject to the above constraints.

Let us now present our numerical results. We consider models with a single condensate and a non-perturbative Kähler potential of the form (2.5). To fix the beta function, we assume that there is one generation of hidden sector matter in the fundamental representation of SU(4). We find that dilaton stabilization, CP violation, and reasonable SUSY

[5]One may argue that observed CP violation may be mainly due to supersymmetric effects in exotic models [25], however such models can hardly be motivated by strings.

[6]Note that this equality cannot be satisfied in the conventional case $m, n > 0$.



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| $\delta_{GS}$ | 0 | 1 | 1.5 |
|---|---|---|---|
| $m$ | $-\frac{1}{15}$ | $-\frac{1}{30}$ | $-\frac{1}{90}$ |
| $n$ | $-\frac{2}{15}$ | $-\frac{2}{30}$ | $-\frac{2}{90}$ |
| $S_{\rm min}$ | 1.75 | 1.71 | 1.69 |
| $T_{\rm min}$ | $1.38 + 0.36i$ | $1.45 + 0.44i$ | $1.33 + 0.33i$ |
| $\varphi_{\rm CKM}$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ |
| $V_0/M_{\rm Pl}^4$ | $1.32 \times 10^{-32}$ | $1.28 \times 10^{-32}$ | $1.42 \times 10^{-32}$ |
| $F_S$/Gev | -2150 | -2120 | -2230 |
| $F_T$/Gev | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $M_a$/Gev | -604 | -608 | -646 |
| $m_\alpha$/Gev | 280 | 276 | 290 |
| $A_{\alpha\beta\gamma}$/Gev | 1690 | 1660 | 1750 |
| $\hat{\mu}$/Gev | 1.75 | 1.73 | 1.82 |
| $\sqrt{B\hat{\mu}}$/Gev | 280 | 276 | 290 |

**Table 1:** Minima and SUSY breaking parameters.

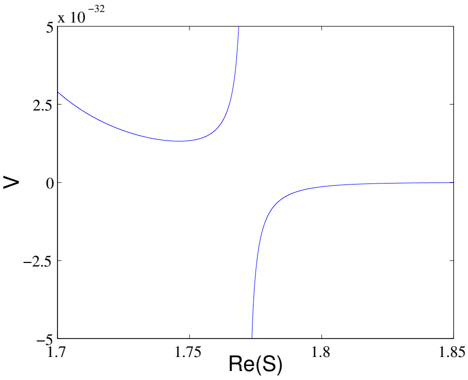


**Figure 1:** Scalar potential with $\delta_{GS} = 0$ and $m = -1/15, n = -2/15$. $T$ is set to its minimum value, $T_{min} = 1.38 + 0.36i$. The minimum in $S$ is at $S_{min} = 1.75$.

breaking can be obtained with, for example, $d = 1$, $p = 10$, $b = 2$, $\delta_{GS} = 0, 1, 1.5$, and $m$ and $n$ given in table 1. We choose $m$ and $n$ such that the modulus gets stabilized at a complex value sufficiently far away from the lines $\mathrm{Im}T = \pm 1/2$ where the CKM phase vanishes [23]. $m$ and $n$ have to increase with increasing $\delta_{GS}$ to produce modulus stabilization: for $\delta_{GS} > \frac{8\pi^2\tilde{\beta}}{3}$, i.e. 1.8 in our case, the superpotential is no longer singular at infinity and $T$ does not settle at a finite value. The choice of the other parameters is dictated by dilaton stabilization and correct SUSY breaking scale. The corresponding numerical results are given in table 1.

The scalar potential for a zero $\delta_{GS}$ is shown in figures 1 and 2. We obtain local minima in $S$ which are separated from the global minima by an infinite barrier. They always lie close to a point where $G_S^S$ vanishes and the scalar potential diverges (figure 1) [18]. Consequently, at the minimum $(G_S^S)^{-1}$ is relatively large (for example, 60 in the $\delta_{GS} = 0$ case). Then, since we need the SUSY breaking scale $F_{S(T)}$ to be around 1 TeV, in realistic

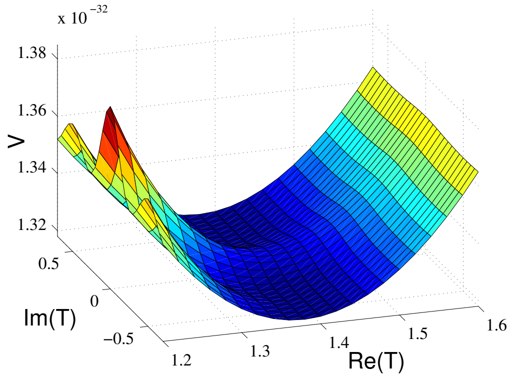


**Figure 2:** Scalar potential with $\delta_{GS} = 0$ and $m = -1/15, n = -2/15$. $S$ is set to its minimum value, $S_{min} = 1.75$. The minimum in $T$ is at $T_{min} = 1.38 + 0.36i$. Note the invariance of the potential under $T \to T + i$.



cases $m_{3/2} = e^{G/2}$ is rather small ($\mathcal{O}(1\,\mathrm{GeV})$) compared to $F_{S(T)}$ as seen from eq. 2.4. The cosmological constant is in these cases positive. Partly due to a large $(G^S_S)^{-1}$, $F_S \gg F_T$ and we have the dilaton domination.

The Jarlskog invariant and the CKM phase can be calculated in orbifold models assuming some fixed point assignment to the MSSM fields. The Yukawa coupling of the states at the fixed points $f_{1,2,3}$ belonging to the twisted sectors $\theta_{1,2,3}$ is given by [27, 28]

$$Y_{f_1 f_2 f_3} = N \sum_{u \in Z^n} \exp\left[-4\pi T\,(f_{23}+u)^T\,M\,(f_{23}+u)\right]. \tag{3.4}$$

where $f_{23} \equiv f_2 - f_3$, $N$ is a normalization factor, and the matrix $M$ (with fractional entries) is related to the internal metric of the orbifold. A complex $T$ does not generally imply a non-zero CKM phase as the Yukawa complex phases may be spurious and eliminated by a basis transformation. This is the case for the prime orbifolds [23] due to restrictive *renormalizable* Yukawa coupling selection rules.[7] In the even order orbifolds, it is possible to produce the CKM phase at the renormalizable level with some favorable fixed point assignment. In table 1, we use a $Z_6 - I$ example of ref. [23] to calculate non-removable Yukawa phases for a given $T$. In all cases they are order one. We, however, do not address the question of the correct fermion mass hierarchy which seems to require non-renormalizable operators [29].

The models we consider possess an "axionic" symmetry

$$S \to S + i\alpha \tag{3.5}$$

with a real continuous $\alpha$. Indeed, the Kähler potential is independent of Im$S$ and the superpotential appears only through $|W|^2$ such that the function $G = K + \log|W|^2$ is invariant under $S \to S + i\alpha$. This symmetry is a consequence of the fact that Im$S$ and Im$T$ have derivative couplings (at least perturbatively) [30]. The symmetry $T \to T + i\alpha$ is

[7] This assumes that the quark fields can be associated with the fixed points rather than their (arbitrary) combinations.

broken by world-sheet instanton effects down to a discrete subgroup, while the $S \to S + i\alpha$ symmetry can only be broken by space-time non-perturbative effects. The latter remains a symmetry of the Kähler potential [31]. The (approximate) invariance of the theory under $S \to S+i\alpha$ allows us to set $S$ and, in the case of dilaton dominated SUSY breaking, $F_S$ real. As a result, the soft SUSY phases are *absent* as required by the EDM constraints. Note, that if $F_T$ were not negligible, the axionic symmetries would not solve the EDM problem due to non-universality of the A-terms [4] and the mechanism suggested in ref. [32] would not work.

As seen from table 1, the soft breaking parameters are all of order a few hundred GeV except for the $\mu$-term. The $\mu$-term is quite small due to $m_{3/2} \ll F_S$ and the fact that the Giudice-Masiero function $Z$ is independent of $S$. We have checked that this remains true if the non-perturbative mechanism for generating the $\mu$-term (see e.g. [33] and [22]) is used. Again, the reason is that the induced $\mu$-term is of order $m_{3/2}$ which is small compared to $F_S$. This seems to be a generic problem in such scenarios unless a different solution to the $\mu$-problem is utilized (e.g. generating $\mu$ through a VEV of a singlet field). It is conceivable that the $\mu$-term receives significant supergravity radiative corrections which may produce $\mu$ of the right size [34]. We note that the above considered mechanisms make it difficult to achieve radiative electroweak symmetry breaking [35].

The other soft breaking parameters have reasonable values. The gaugino masses and the A-terms are dominated by the $F_S$ contributions, while the scalar masses and $B\mu$ receive the dominant contributions from $V_0$. We note that at the tree level $V_0$ coincides with the cosmological constant, whereas at the loop level $V_0$ and the true cosmological constant receive different quadratically divergent corrections [36]. Thus, a non-vanishing $V_0$ does not imply that the cosmological constant is non-zero. In fact, in our case $V_0 \sim \tilde{m}^2 M_{\rm Pl}^2$ with $\tilde{m}$ being the typical soft breaking mass. This is exactly of the order of the quadratically divergent 1-loop corrections [36] which can be of either sign depending on the specifics of the model. Thus, with an appropriate choice of the hidden sector it is possible to cancel the cosmological constant.

We find that other choices of $d, p$ and $b$ give broadly similar results and our conclusions apply quite generally.

## 4. Conclusions

We have studied the possibility of obtaining realistic vacua in heterotic string models possessing the SL$(2, Z)$ modular invariance. We find that dilaton stabilization, realistic CP violation and acceptable SUSY breaking can be obtained if (1) non-perturbative Kähler potential is used to stabilize the dilaton, (2) the superpotential is singular at the fixed points of the modular group.

Our essential result is that it is possible to reconcile large CP violation in the Standard Model and small CP violation in the soft SUSY breaking terms. This necessitates dilaton dominated supersymmetry breaking and an axionic symmetry $S \to S+i\alpha$ (which is natural in our class of models). The only phenomenological difficulty in this case is a small tree-level $\mu$-term. This problem may potentially be rectified by incorporating quadratically divergent radiative corrections [34]. The same applies to the cosmological constant. Of course, it remains a challenge to obtain the desired properties from explicit string models.

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